Time-Surface Maps
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1 INTRODUCTION
While topological analysis of stationary flow is mathematically well defined by analysis of critical point graphs, topology of three-dimensional unsteady flow fields is still a challenging task and subject of ongoing research. Research on topology analysis of unsteady flow often focuses on ways to transfer stationary concepts to time-varying data sets or use localized Lagrangian flow analysis to give a "topological" view of the whole data set, resulting in the extraction of stable and unstable manifolds. Visualizations of these manifolds provide experts with an insight into flow behavior in a finite-time interval that allows the drawing of conclusions about flow divergence and mixing, while lacking a clear visualization of input-output behavior of the complete dynamical system. We contribute a technique to extract, analyze, and visualize properties of this input-output behavior by using topological analysis of time-surface statistics to model such flow "transfer functions". Inspired by the concept of impulse response analysis of dynamical systems, we define time-surfaces located at flow sources as the analogon of a peaked impulse function in flow fields. Statistical properties of these adaptive time-surfaces are accumulated during advection and mapped into surface parameter space, as the surface leaves the data set or vanishes in a sink. Topological analysis of these scalar maps clusters time-surface parameter space into distinct regions, effectively segmenting cycle-free flow fields into homogeneous regions. We show results that demonstrate how visualizations of the produced region geometry can aid in understanding instantaneous transfer behavior of flow fields.

2 TIME-SURFACE STATISTICS
A time-surface $T : \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R}^3$ of a time-varying three-dimensional flow field $f : \mathbb{R}^3 \times \mathbb{R} \rightarrow \mathbb{R}^3$ at time $t$ is defined as a function

$$T(u,v;t) = T(u,v;t_0) + \int_{t_0}^{t} f(T(u,v;\tau);\tau) \, d\tau$$

(1)

where $T(u,v;t_0)$ describes the initial surface over parameter space $(u,v) \in \Omega \subseteq \mathbb{R}^2$. In practice, a time-surface is computed by sampling $\Omega$ at time $t_0$ with particles $p(u,v)$. In an arbitrary later time-step, $T$ then consists of the connected set of advected particle positions. Recent advances in integral surface extraction [1] make it possible to extract and model adaptive time-surfaces in an efficient manner by inserting new surface particles at later time steps to avoid degenerate and ill-shaped triangles in the corresponding time-surface mesh and adjust surface resolution to capture small scale flow features. These refinement steps generally lead to a non-uniformly sampled parameter space and allows sufficiently dense particle tracing without oversampling the flow field.

To capture characteristics of the transfer function of a flow field, we create a time-surface at inflow [2] and source locations and advect it through the flow field, similar to methods used in stream function computation [3] and time-surface distortion analysis in steady flow [5]. During particle advection different statistics are accumulated on a per-particle basis, such as:

- path-line length
- average/max/min speed
- flow gradient
- particle life time
- relative particle position
- flow vorticity
- surface curvature

After advection (surface vanishes or leaves the data set), these quantities are mapped into surface parameter space, where they represent a set of two-dimensional scattered scalar fields. From a given single such time-surface map we compute a continuous representation by the use of standard scattered data approximation techniques. An additional statistical property we are interested in is the particle density in $\Omega$, where high values indicate surface-parallel strain or increased surface stretching. Analysis and evaluation of these statistics aims to answer the question of what happens to a continuous front as it is transformed by the dynamical system described by the flow field.

On these time-surface maps, topological methods of scalar field analysis facilitate identification of regions with homogeneous properties and discontinuities. We perform this topological analysis on a discretized version by sampling the time-surface map into a normalized two-dimensional image $I$. A approximation of how this image is segmented into distinct cells by the unstable and stable Morse complex is obtained by performing a watershed segmentation on $I$ and $1-I$. Individual cells of the complex are thus bounded by extremal features of the image. The intersection of both subdivisions yields an approximation of the Morse-Smale complex [4]. Depending on the choice of the analyzed time-surface statistic, extremal values represent different aspects of flow behavior. Extremal values of the spectral norm of accumulated flow gradients are comparable to FTLE ridges and highlight flow divergence, whereas extrema in particle position maps and vorticity data provide general data about mixing.

Each such cell $c \subseteq \Omega$ implicitly represents a path volume defined by the union of all path-lines of particles in $c$. By grouping of path-lines obtained during time-surface advection and identification of boundary path-lines of individual cells, we are able to construct accurate path-volume meshes directly from the available scattered particle set. This construction has several advantages over the extraction of isosurfaces in stream-function space, as it avoids expensive scattered data interpolation and resampling of the data set. During triangulation, we solve several challenges posed by the nature of available particle data (scattered in $\Omega$ and time due to time-surface adaptivity). The final decomposition of the data set into (possibly overlapping) path-volumes can provide insights into various mixing properties of the underlying flow field as well as highlight flow regions with extremal behavior.
3 RESULTS AND CONCLUSION

For visualization purposes we make use of standard mesh shading techniques as well as gradient-based volume rendering of the resulting path-volume geometry to provide clear looks at volume features and reduce occlusion. Figure 1 shows an example of particle positions in $\Omega$ and a selection of resulting continuous time-surface map representations as well as a watershed segmentation of the density map. In Figure 2, we show resulting path-volumes for a basic example of sinusoidal flow in different visualization techniques. Topological analysis of the particle density map allows extraction of path-volumes crossing flow regions with maximal or minimal time-surface stretching. Flow segmentation based on maximal particle advection time is shown in Figure 3, where particles at no-slip boundaries are destroyed upon arrival, thus splitting the data set into two regions. In contrast to path-volumes in stationary data sets, extracted volumes in time-dependent data sets can (self) intersect. Visualization problems caused by mesh intersection can be solved by the use of volume rendering techniques (see Figure 4).

Figure 1: Different rendering of mappings of time-surface particles to parameter space. (1) Discrete particle positions in $\Omega$, (2) continuous density representation, (3) vector map of final particle positions and (4) cells of unstable Morse complex for particle density map of the time-surface shown in Figure 2.

Figure 2: Time-surface map and a selection of two path-volumes. Volume-rendering of path-volumes (left) and solid mesh triangulation with two time-surface snapshots (right). The particle density map assumes maximal values, where time-surface stretching is extremal.

We have shown how topological analysis of time-surface statistics can be used to extract flow volume geometry that segments the flow field according to prevalent features in its input-output behavior. This allows analysis of instantaneous input transformation in stationary and time-varying vector fields. To yield a fully time-varying analysis of a flow field, this method may be applied to a number of sequential time-steps. In contrast to existing unsteady flow visualization techniques in the field of Lagrangian Coherent Structures, this work is a step towards statistical flow transfer function analysis rather than an attempt to provide a dense (in space and time) and direct analysis of flow hyperbolicity.

REFERENCES