The LIR Domain Decomposition System applied to Cartesian Grids

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We introduce a novel multi-dimensional domain decomposition method. A new type of tree combines the advantages of the octree and the KD-tree without having their disadvantages.

The tree structure is defined by a topological algebra based on the basic symbols $A = \{L, I, R\}$ that encode the decomposition steps:

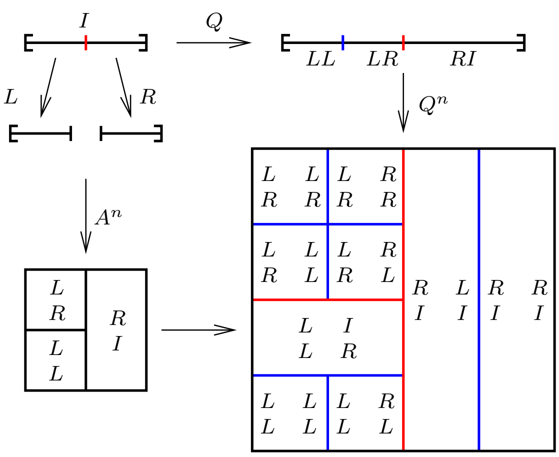
- L : Decomposition to the **left**
- I : No Decomposition - **identity**
- R : Decomposition to the **right**

Successive applications of basic symbols such that the sequences have the form

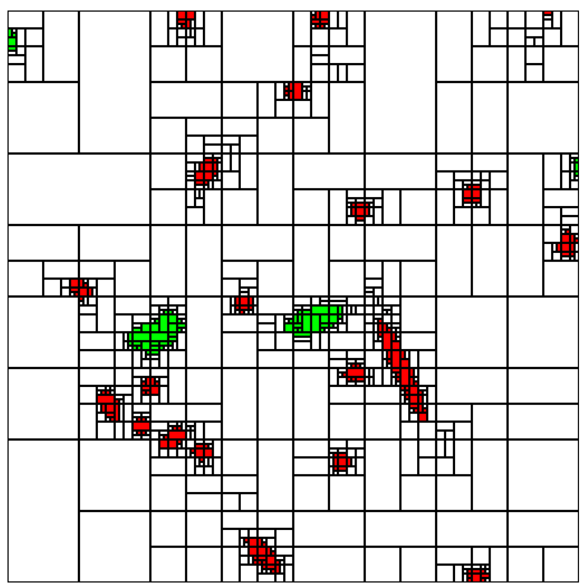
$$Q = \{(q_i \in A)_{i\in\mathbb{N}} : \forall_{i\in\mathbb{N}} q_i = I \Rightarrow q_{i+1} = I\}$$

yield a unique and minimal representation of subdomains. Vectors of sequences (or sequences of basic vectors) allows multi-dimensional decomposition of domains. The set of successors is restricted such that each cell has the partition of unity property to decompose domains without overlap. The data structure allows local refinement, parallelization and proper restriction of transition ratios between cells.

The LIR-tree has the interior-cell complexity of the octree and the leaf-cell complexity of the KD-tree. Thus, we have less than half the number of cells compared to the other tree structures in the three-dimenional case. The LIR-Tree is used as a basic framework to represent scientific data efficiently. In future work we want to investigate the abilities of the LIR domain decomposition system in numerical simulations.



Recursive and dimensional generalization of A.



A fiber structure represented by the LIR-Tree.